Robust spatial approximation of laser scanner point clouds by means of free-form curve and surface approaches

IUGG, IAG Symposia 2015
G05 GNSS++: Emerging Technologies and Applications
Prague, June 24, 2015

Johannes Bureick, Hamza Alkhatib and Ingo Neumann
Geodetic Institute, Leibniz Universität Hannover, Germany
**Advanced Rail Track Inspection System**

**Key figures**
- Speed: 1 m/s
- Frequency scanner: 200 Hz
- Frequency tracker: 1000 Hz
- Point distance (along): < 5 mm
- Point distance (across): < 0.22 mm
Motivation

- Uncertainty requirements
  - Position and height of the rail track < 0.5 mm
  - Surface defects < 0.2 – 0.5 mm
- Data characteristic
  - Profile-wise kinematic measurements
  - Data gaps
  - 3D representation
- Algorithms / Parametrization
  - Approximate point cloud
  - Identify deformations
  - Cope with uncertainty requirements and data characteristic
• Motivation
• Mathematical basics – approximation
• Modification parameters / Related work
• Methodology
• Results
• Summary / Outlook
Parametric curve approximation: B-Spline

- Functional relation: piecewise polynomial function
\[ x(u) = \begin{bmatrix} x(u) \\ y(u) \end{bmatrix} = \sum_{i=0}^{n} N_{i,p}(u) p_i, \quad u_{\text{min}} \leq u < u_{\text{max}} \]

- Curve-point \( x(u) \)
- Basis function \( N_{i,p}(u) \)
- Control point \( p_i \)
- Location parameter \( u \)
Mathematical basics - approximation

- $N_{i,p}(u)$: basis function of degree $p$; recursive function:

$$N_{i,0}(u) = \begin{cases} 1, & u_i \leq u < u_{i+1} \\ 0, & \text{else} \end{cases}$$

$$N_{i,p}(u) = \frac{u-u_i}{u_{i+p}-u_i} N_{i,p-1}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}} N_{i+1,p-1}(u)$$

- Location parameter $u$:

$u_i \leq u < u_{i+1}$

- Knotvector $U = [u_{min}, ..., u_{min}, ..., u_i, u_{i+1}, ..., u_{max}, ..., u_{max}]$

  - Parameter: Control points $p_i$
  - Observations: Measured laserscanner points $x(u)$
  - Design matrix: Consists of basis functions $N_{i,p}(u)$
- Determination of basis function degree and number of control points
    - Information criterion
    - Significance test
  - Location parameter $u$ of the measured points
    - Choice (cf. Piegl and Tiller 1997)
      - Equally spaced
      - Chord length
      - Centripetal method
    - Estimation (cf. Lai and Lu 1996)
Modification parameters / Related work

- Determination of knotvector
  - Alignment to location parameter (cf. Piegl and Tiller 1997)
  - Alignment to curvature of measured points (cf. Park and Lee 2007)
  - Estimation (cf. Schmitt and Neuner 2015)
- Our approach:
  - Monte-Carlo-Method with probabilities depending on location and curvature
- Estimation of control points
  - Estimator
    - Least squares (cf. Koch 2009)
    - M-estimator (Huber-, Hampel-)
    - L1-norm-estimator
  - Random sample consensus algorithm (RANSAC)
Methodology MCM

**Start** => Choose:
- Number of control points $n$
- Degree $p$ of basis function

1. Calculate a probability depending on location and curvature
2. Randomly choose the $n-p-1$ internal knots
3. Arrange the complete knotvector $U$
4. Estimate control points (GMM)
5. Calculate residual sum of squares $\Omega$

**Yes**
- Store the results for $U$, the control points and $\Omega$

**No**
- Check if $\Omega$ is smaller than the best $\Omega$?
- Stop-criterion reached?

**End**
Results
Results

Degree: $p = 2$
Control points: $n = 100$
Results

$\Omega_{\text{Basic}} = 12.75 \text{ mm}^2$

$\Omega_{\text{MCM}} = 1.08 \text{ mm}^2$

ca. 0.4 mm

> 0.5 mm
Results

- **MCM**
- **Basic**

![Graphs showing MCM and Basic iterations](image)

- Absolute residuals $|\mathbf{v}|$ [mm] for MCM and Basic iterations.
- Iteration count for MCM and Basic.
- Graph of $\Omega$ versus iteration.
For same number of parameters MCM obtains significant better results
  - Especially when data gaps occur
- Computational cost is higher
- Applicable for modelling of the local earth gravity field?

- Extension to B-Spline surfaces
- Integration of further prior knowledge
  - CAD-Modell
  - Previous profiles / measurements
Thank you for your attention.

Robust spatial approximation of laser scanner point clouds by means of free-form curve and surface approaches

M.Sc. Johannes Bureick, Dr.-Ing. Hamza Alkhatib and Prof. Dr.-Ing. Ingo Neumann
Geodetic Institute, Leibniz Universität Hannover, Germany
bureick@gih.uni-hannover.de | www.gih.uni-hannover.de


